



# **A STUDY ON INFLUENCE ANALYSIS OF TWO TYPES OF BIASED ESTIMATORS IN LINEAR REGRESSION MODELS**

**A Dissertation**

**By**

**ABOOBACKER JAHUFER**

**DOCTOR OF PHILOSOPHY (Ph.D)  
IN  
STATISTICS**

**May 2010**

University Code: 10384  
Student Code: 15420070154132

UDC \_\_\_\_\_

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**A Study on Influence Analysis of Two Types of Biased  
Estimators in Linear Regression Models**

线性回归模型中两类有偏估计的影响分析研究

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***Dedicated to My Beloved  
Parents and Family***

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**The Author**



# 摘要

在过去的两世纪多，普通最小二乘法（OLS）得到了广泛的发展和应用，与之相关的研究领域包括物理学、医学、生物学、地质学、工程学、经济学、管理学和社会学等。自该方法使用以来，人们就普遍认为基于最小二乘法的统计推断对异常值具有敏感性，因此，诊断技术（如残差图等）被发展用来探测这些异常值和检测模型的假设是否正确等。

一般来讲，样本中的不同数据集会对拟合回归模型产生不同的影响。这也就是说，参数估计或者预测有可能更多的依靠样本中的小部分而不是大多数样本。这就需要指出这些对模型具有较强影响的样本点，并估算这些样本对模型的影响。如果这些具有较强影响的样本点本身存在问题，我们应该加以删除；另一方面，若这些点本身并没有什么问题，那么有必要对所建立的模型进行修正。

在线性回归建模过程中，当回归自变量间具有多重共线性时，尽管相应回归参数的 OLS 估计量仍然是最佳线性无偏估计(BLUE)，但是多重共线性却会使其产生很大的方差，从而导致其均方误差也很大，其后果在于相应的统计推断并不可靠。为了克服多重共线性带来的后果，人们在大量文献中提出了约束最小二乘、混合估计和岭型有偏估计等。

作为解决多重共线性问题并且发展较为迅速的岭型有偏估计，具有很强的思想性和实用性，使用较为广泛，尤其是在计量经济学，工程技术和其他相关领域，岭型有偏估计分析已经为大量新理论、技术、思想和方法的发展打下了坚实的基础。

到目前为止，尽管不少岭型有偏估计量都可以用来拟合具有多重共线性数据的回归模型，但人们对这些有偏估计的影响分析方法并没有做出充分的研究。很多研究者都曾试图证明“观测值对岭型有偏估计的影响与其对相应 OLS 估计的影响有很大差异，岭型有偏估计可以冲淡异常数据对多重共线性的影响”，那么，结果到底如何呢？

本文试图在自变量存在多重共线性的条件下，将总体影响分析（Global Influence Analysis）、局部影响分析（Local Influence Analysis）、Cook 的最小扰动影响分析（Cook's Minor Perturbation Influence Analysis）以及 Box-Cox 变换影

响分析 (Box-Cox Transformation Influence Analysis) 等四种影响分析方法作进一步的发展, 以便用于讨论两类有偏估计中的强影响观测值, 这两类有偏估计分别是调整岭回归估计 (Modified Ridge Regression Estimator, MRRE) 和 Liu 估计。

在全局影响分析中, 我们讨论了基于 MRRE 和 Liu 估计的残差、杠杆值、DFFITS 值以及 Cook 距离的两种形式, 并导出了 DFFITS 和 Cook 距离的两种形式甄别影响点的近似删除公式。在局部影响分析中, 我们通过对模型中的方差, 解释变量进行扰动以及适当的选择岭参数估计, 得到了两类有偏估计的广义 Cook 影响度量; 在 Cook 最小扰动影响分析中, 我们分别对误差方差是已知和未知的情况, 讨论了两类有偏估计的影响分析。最后, 在误差方差扰动的情况下, 对 Box-Cox 变换模型的两类岭估计进行了影响分析研究。

为了具体说明 MRRE 和 Liu 估计的总体影响分析方法、局部影响分析方法、Cook 最小扰动影响分析方法以及 Box-Cox 转化影响分析方法, 本文利用两组宏观经济数据进行了实例分析。

**关键词:** 调整岭回归估计; Liu 估计; 影响分析; 多重共线性; 强影响观测值

## ABSTRACT

In the past two centuries, the ordinary least squares (OLS) method has in use. It is being developed and applied in many research fields, the related research fields are Physics, Medicine, Biology, Geology, Engineering, Economics, Management Science, Sociology and so on. From the beginning, it was generally believed that the statistical inference based on least squares is sensitive to unusual data. Therefore, some diagnostic techniques (for example, residual plot) were developed to identify anomalous observations and check violations of model assumptions.

In general, different subsets of the data may produce different influence on the fitted regression model. That is, parameter estimates or predictions may depend more on the influential points than on the majority of the data. It would be liked to locate these influential points and assess their impact on statistical influence of the model. If these influential points are bad values then they should be eliminated. On the other hand, there may be nothing wrong with these points, but if they control key model properties, as they would like for them to, they could affect the use of the model. Therefore, it is necessary to modify the model.

In the linear regression model building process, if there exists multicollinearity among the regressors, the high variance of OLS estimator can make the corresponding mean squared error (MSE) very large although, it is still a best linear unbiased estimator (BLUE). This may lead to the statistical inference unreliable. To overcome the multicollinearity problems, the restricted least squares (RLS), mixed estimators and ridge type biased estimators methods are given prominence in literature.

The ridge type biased estimator models have become an ideal, popular and useful technique in the path of its fast growth in the world of knowledge to solve multicollinearity problems. Also the recent literature, particularly touching upon the fields of econometrics, engineering and other statistical areas the ridge type biased estimators analysis, have come to lay concrete foundation to produced a number of new theories, techniques, ideas and methods.

Although many ridge type biased estimators have been proposed to fit the regression model for multicollinearity data, so far, very few related methods on influential analysis for ridge type biased estimators have been developed. Many authors have attempted to prove that “the influence of the observations on ridge type biased estimators is different from that of the corresponding OLS estimator. Whether this statement is true or false?

In this research study, it is aimed to further develop influence analysis methods including global, local, Cook's minor perturbation and Box-Cox transformation to detect influential observations in ridge type biased estimators namely modified ridge regression estimator (MRRE) and Liu estimator when these two estimators are used to mitigate the effects of multicollinearity is existing among the regressors.

Under the global influential analysis residual, leverage, DFFITS and two versions of Cook's distance are studied for MRRE and Liu estimator. In addition to these influential analyses for both ridge type estimators, approximate deletion formulas are proposed for detection of influential points using DFFITS and two versions of Cook's distance. Moreover, under the local influential analysis variance perturbation, perturbation of individual explanatory variable and an assessment of biasing parameter are analyzed for both biased estimators. Under the Cook's minor perturbation influential analysis variance perturbation when variance of error is known and unknown situations and perturbation of individual explanatory variable are studied for both ridge type estimators. Finally under the Box-Cox transformation variance perturbation is studied for both biased estimators.

To illustrate the methodologies developed under the global, local, Cook's minor perturbation and Box-Cox transformation influential analysis for MRRE and Liu estimator, two real macroeconomic data sets are used.

**Key Words:** Modified Ridge Regression Estimator; Liu Estimator; Influence Analysis; Multicollinearity; Influential Observation.

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